Value Magnitudes and the Kahn Employment Multiplier

by

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1. Introduction

The main thrust of the Sraffian critique of Marx's economics has been to demonstrate the irrelevance of value magnitudes. Even the Fundamental Marxian Theorem, that positive surplus value is necessary and sufficient for the existence of positive profits, has been falsified. Steedman (1977, p. 154) showed, with additive value accounts and a two sector example, that it is quite possible to establish positive money profits together with negative surplus value. This is made possible by the derivation of a negative labour value for one of these sectors, due to the impact of joint production (the prevailing technology in capitalist production). As King (1982, p. 159) commented: 'If this condition fails, it is evident that something is seriously wrong with Marx's analysis.'

In an attempt to rationalise this negative surplus value outcome, labour value magnitudes have been interpreted as employment multipliers. Steedman (1977, p. 158) invoked Morishima's observation that 'values are not more than the employment multipliers discussed by Kahn and later by Keynes…' With sectors producing a mix of different commodities it is quite feasible that output can increase with a reduction in employment: a negative employment multiplier. There has, however, been little recognition that the employment multiplier might be related to Marx's wider political economy. In the Sraffian critique of Marx, an attempt has been made to isolate the labour theory of value from Marx's system. Steedman referred to 'Marx's many insights which were independent of his value magnitude reasoning'. The Sraffian critique 'leaves open for investigation all the difficult problems relating, for example, to money, to effective demand and to crises…' (*ibid*, p. 206). A notable exception is provided by Kurz (1985), who considered effective demand and labour embodied categories, but nevertheless argued that 'there is not such thing as "the" multiplier' (p. 134).

Of central importance to the theory of effective demand is the Kahn (1931) employment multiplier, which was developed in the U.K. during the Great Depression, forming the centrepiece of Keynes's *General Theory* (1936). The basic idea, that an increase in primary employment will have a multiplied effect on total employment, has been a clarion call for state intervention to boost employment. The purpose of this paper is to examine the structure of the Kahn employment multiplier and its relationship to value magnitudes. An established though not widely known discovery in Marxian economics is the constituent role of surplus value in the closed input-output system (Dixon 1988, Olgin 1992, Trigg 2006). The analytical contribution of this paper is to derive this result under joint production technology. A derivation of the Kahn multiplier relationship in the first part of the paper is followed in the second by an exploration of its structure from a Marxian perspective.

2. The Kahn Employment Multiplier

Consider a closed physical system with m sectors, in which all capital is used up during the production period and there is pure joint production. Let $\mathbf{A}$ be a square matrix of interindustry technical coefficients, $\mathbf{l}$ a vector of direct labour coefficients, $\mathbf{b}$ a vector of subsistence consumption coefficients for workers, $\mathbf{X}$ a vector of gross outputs, and $\mathbf{f}$ a vector representing final demands. To model joint production, a square output matrix $\mathbf{B}$ is also defined in which some of the off diagonals are non-zero. There are still only m commodities produced in total with this model, but each sector may produce more than one of these commodities. The input-output system takes the form:

$$\mathbf{BX} = \mathbf{AX} + \mathbf{blX} + \mathbf{f} \qquad (1)$$

This input-output system can be re-interpreted in terms of net outputs. A vector of net outputs can be defined as $\mathbf{Q} = (\mathbf{B} - \mathbf{A})\mathbf{X}$, and under the assumption that $(\mathbf{B} - \mathbf{A})$ is non-singular it follows that $\mathbf{X} = (\mathbf{B} - \mathbf{A})^{-1}\mathbf{Q}$. Hence, (1) can be re-expressed as

$$\mathbf{Q} = \mathbf{bl}(\mathbf{B} - \mathbf{A})^{-1}\mathbf{Q} + \mathbf{f} \qquad (2)$$

With $\boldsymbol{\lambda} = \mathbf{l}(\mathbf{B} - \mathbf{A})^{-1}$ defined as the vertically integrated labour coefficients under pure joint production:

$$\mathbf{Q} = \mathbf{b}\boldsymbol{\lambda}\mathbf{Q} + \mathbf{f} \qquad (3)$$

Now if we multiply throughout by $\boldsymbol{\lambda}$ it follows that

$$\boldsymbol{\lambda}\mathbf{Q} = \boldsymbol{\lambda}\mathbf{b}\left[\boldsymbol{\lambda}\mathbf{Q}\right] + \boldsymbol{\lambda}\mathbf{f} \qquad (4)$$

such that

$$L = \frac{\mathbf{1}}{\mathbf{1} - \boldsymbol{\lambda}\mathbf{b}}\boldsymbol{\lambda}\mathbf{f} \qquad (5)$$

Equation (5) is a multisectoral multiplier framework in which $k = (1/1 - \boldsymbol{\lambda}\mathbf{b})$ is a scalar employment multiplier specifying the relationship between total employment $L = \boldsymbol{\lambda}\mathbf{Q}$ and the direct and indirect labour required to produce the surplus product/final demand $(\boldsymbol{\lambda}\mathbf{f})$. This is comparable to the original employment multiplier devised by Kahn (1931), with $L_I = \boldsymbol{\lambda}\mathbf{f}$ representing primary employment and $L - L_I$ representing secondary employment.

From a Marxian perspective, the key insight, demonstrated here for the general case of joint production, is that the denominator of this multiplier is a term representing Marx's category of surplus value. Since $\boldsymbol{\lambda}\mathbf{b}$ represents the labour embodied in worker consumption per unit of labour (the value of labour power), the denominator $1 - \boldsymbol{\lambda}\mathbf{b}$ is a term representing the amount of surplus value extracted per unit of labour (see Trigg 2006). The structure of the Kahn employment multiplier depends directly on the extraction of surplus value.

3. The Surplus Value Condition

This multiplier demonstrates the analytical power of surplus value in its embodied labour form. Assume that a proportion of each unit of labour power is extracted as surplus value:

$$0 < 1 - \boldsymbol{\lambda}\mathbf{b} < 1 \qquad (6)$$

A number of key results can be established from this surplus value condition. First, it follows from (6) that $k > 1$: with a multiplier higher than 1 the Kahn multiplier ensures that primary employment induces a higher volume of total employment $(L > L_I)$.

This result contrasts with the conclusion of Kurz (1986, p. 130) that 'an increase in primary employment, could be associated with a *decrease* in the volume of total employment' (original emphasis). The key difference is the way in which total employment is determined. Kurz derives a matrix multiplier $\mathbf{M}$ to determine net output[1]:

$$\mathbf{Q} = \mathbf{M}\mathbf{f} \qquad (7)$$

And from this relationship total employment is defined by

$$L = \boldsymbol{\lambda}\mathbf{M}\mathbf{f} \qquad (8)$$

As before, primary employment is defined by

$$L_I = \boldsymbol{\lambda}\mathbf{f} \tag{9}$$

By modelling primary and total employment using separate equations, Kurz creates the possibility of a disjuncture between the two macroeconomic quantities. Say via (7) a particular physical composition $\mathbf{f}_1$ results in higher total employment than a different physical composition $\mathbf{f}_2$. In contrast, via (8), it is possible for $\mathbf{f}_1$ to produce a lower volume of primary employment than $\mathbf{f}_2$. An increase in primary employment does not guarantee an increase in total employment. For Kurz (1986, p. 501) the Kahn multiplier measure is 'not unequivocal'.

In contrast, the Kahn multiplier defined in (5) provides an unequivocal expansionary relationship between primary and secondary employment, once surplus value is extracted under condition (6). To use a term borrowed from Pasinetti (1981), this multiplier relationship is 'genuinely macroeconomic': invariant to the physical composition of final demand.

Our second result concerns the occurrence of negative labour values, as demonstrated by Steedman (1977) under joint production. It is mathematically possible in (9) for negative elements of $\boldsymbol{\lambda}$ to generate a negative total quantity of primary employment. The derivation in equations (1) to (5) accounts for this possibility, thereby enhancing the genuinely macroeconomic structure of the Kahn multiplier. The structure of the multiplier remains unchanged by negative labour values. For an economically feasible solution, in which $L > 0$, the surplus value condition (6) also ensures a positive volume of primary employment: $L_I > 0$. This further demonstrates the analytical power of the embodied surplus value category. Even under joint production, with negative labour values, the surplus value condition is critical to the definition of the Kahn multiplier relationship.

Finally, a similar result can be established under a mathematical scenario considered by Kliman (2001). In his critique of 'physicalism', elements of the final demand/physical surplus value vector $\mathbf{f}$ are allowed to be negative, which via (9) could result in a negative volume of primary employment. However, our surplus value condition (6) also ensures that this scenario is not economically feasible – it would through (5) generate negative total employment.[2] This provides further proof that embodied surplus value has a key role to play in defining an economically viable Kahn multiplier relationship.

3. Conclusions

The contribution of this paper has been to derive a scalar Kahn employment multiplier from an input-output system with pure joint production. It is usual in macroeconomics to specify an employment multiplier that is positive and more than 1 in magnitude. Our derivation shows that Marx's category of surplus value, as defined in labour embodied terms, is critical to this specification. For a surplus value producing economy, a genuinely macroeconomic multiplier relationship has been defined that is invariant to changes in the either the physical composition of final demand, negative labour values or negative physical surplus elements. Far from being independent of the wider study of effective demand, value magnitudes are core to the structure of one of its key analytical tools: the Kahn employment multiplier.

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Footnotes

[1] Kurz (1986, pp. 123-126) derives this multiplier matrix using both quantity and price systems. It depends, amongst other things, on the rate of profit and the consumption propensities of capitalists and workers (see Trigg 2006, p. 29, for a consideration of capitalist consumption). Using (3), a simplified equivalent to this matrix in our quantity system would take the form $\mathbf{M}^{*} = (\mathbf{I} - \mathbf{b}\boldsymbol{\lambda})^{-1}$, where $\mathbf{Q} = \mathbf{M}^{*}\mathbf{f}$.

[2] We also discount the trivial solution to (5) in which a zero physical surplus could generates zero primary and total employment.